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Advanced Discrete Mathematics Problem Solving / Questions 1

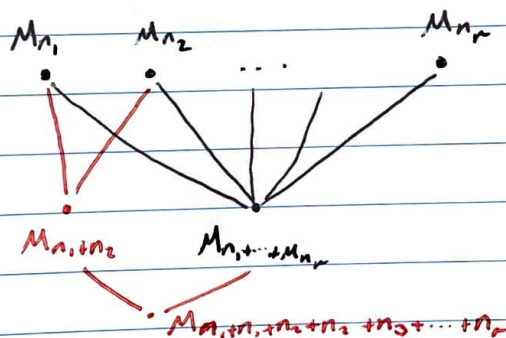
Source of Bratteli diagrams

$M_n(\mathbb{C})$ is an algebra consisting of matrices with entries in $\mathbb{C}$.

$$M_n(\mathbb{C}) \oplus M_m(\mathbb{C}) = \left\{ \begin{pmatrix} A & \{ \}^n \\ \{ \}^m & B \end{pmatrix} \mid A \in M_n(\mathbb{C}) \text{ and } B \in M_m(\mathbb{C}) \right\}$$

Notice that $M_{n_1}(\mathbb{C}) \oplus \dots \oplus M_{n_r}(\mathbb{C}) \subseteq M_{n_1 + \dots + n_r}(\mathbb{C})$.

We can draw this inclusion of matrix algebras, say, as follows:



We can also fill in other elements.

The 'limit' of this tower is an infinite-dimensional algebra which perhaps ought to be called a matrix algebra but is instead called a subfactor.

This is the main object of study in the field of C^* -algebras.

Correspondences

Young $\leftrightarrow$ group algebra of the symmetric group

Brauer $\leftrightarrow$ Brauer algebra

Temperley-Lieb $\leftrightarrow$ Temperley-Lieb algebra

Pascal $\leftrightarrow$ Planar rook monoid algebra

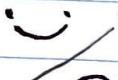
Brauer Algebra

B_n : connecting k dots to k dots.

$B_1(n)$ 

$B_2(n)$   

$B_3(n)$     

Multiplication is joining the diagrams and removing circles while simultaneously multiplying by n for each circle.

Temperley-Lieb Algebra

The Temperley-Lieb algebra is the planar Brauer algebra.
This means we only consider elements with 0 crossings.

TL, :

TL_2 : $\begin{array}{c} | \\ | \end{array} \begin{array}{c} | \\ | \end{array} \begin{array}{c} \cup \\ \cup \end{array}$

TL_3 :