

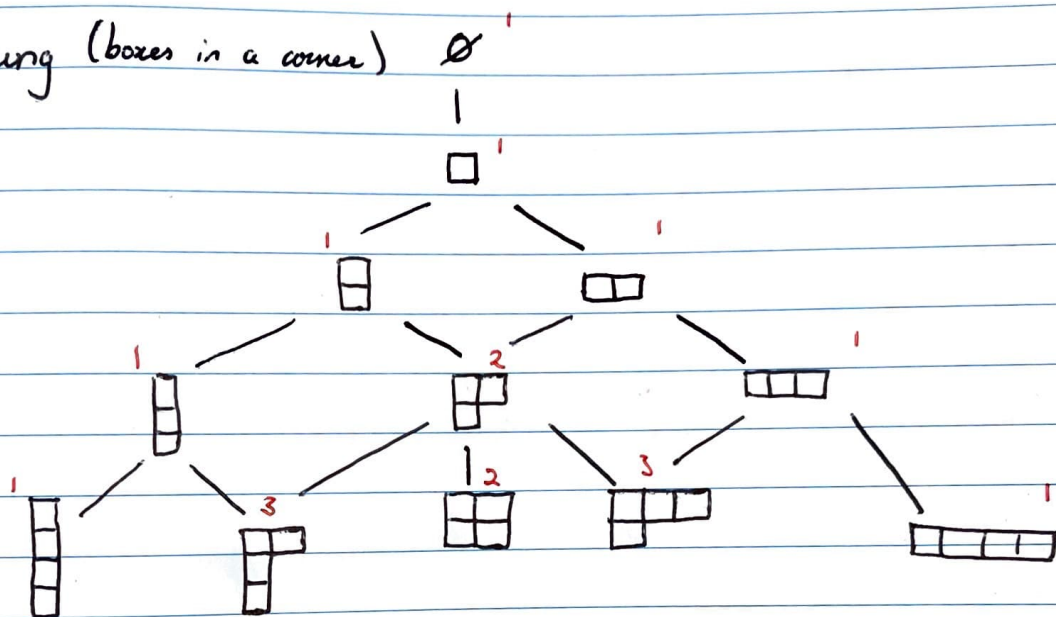
Advanced Discrete Mathematics

Lecture 1

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Bratteli Diagrams

Young (boxes in a corner)



Next to each picture, mark the number of paths to that picture starting from $\emptyset$ in red.

For each row, find the sum of squares of these red numbers.

We have:

$$(0) 1^2 = 1$$

$$(1) 1^2 = 1$$

$$(2) 1^2 + 1^2 = 1 + 1 = 2$$

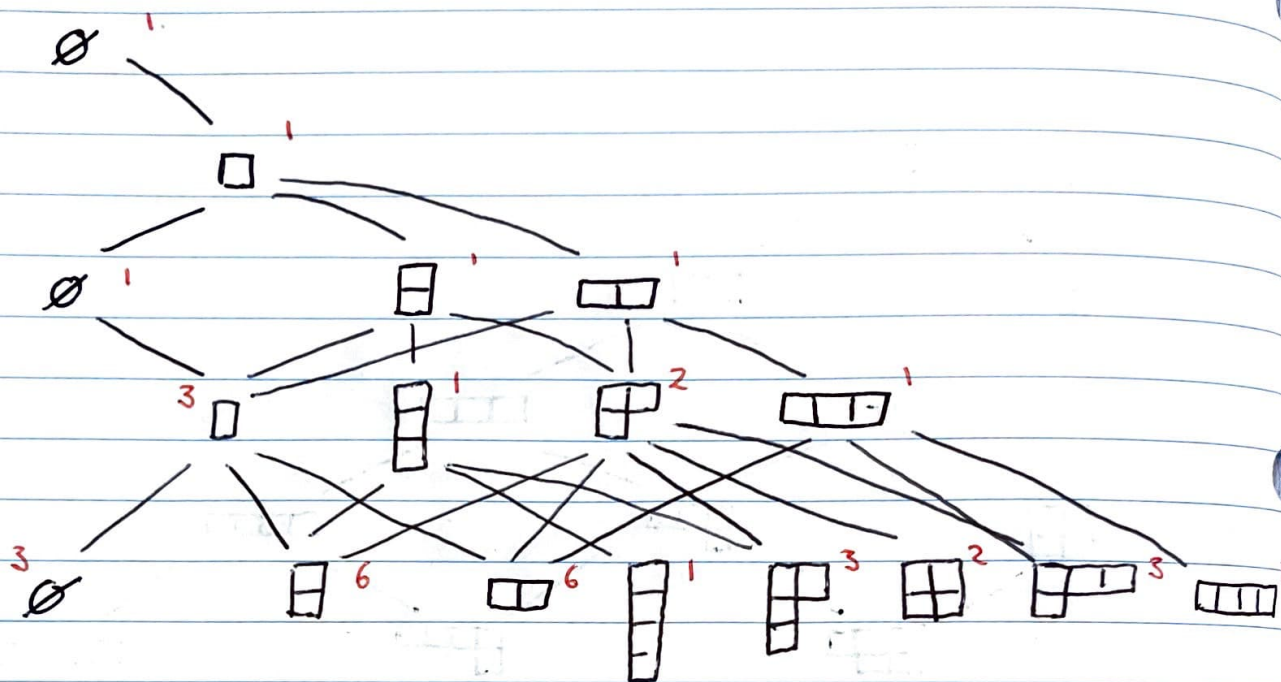
$$(3) 1^2 + 2^2 + 1^2 = 6$$

$$(4) 1^2 + 3^2 + 2^2 + 3^2 + 1^2 = 24$$

1, 1, 2, 6, 24, ...

These are the factorial numbers: $k!$

Braker (add and remove)



We have:

(0) $1^2 = 1$

$$(1) \quad 1^2 = 1$$

$$(2) \quad 1^2 + 1^2 + 1^2 = 3$$

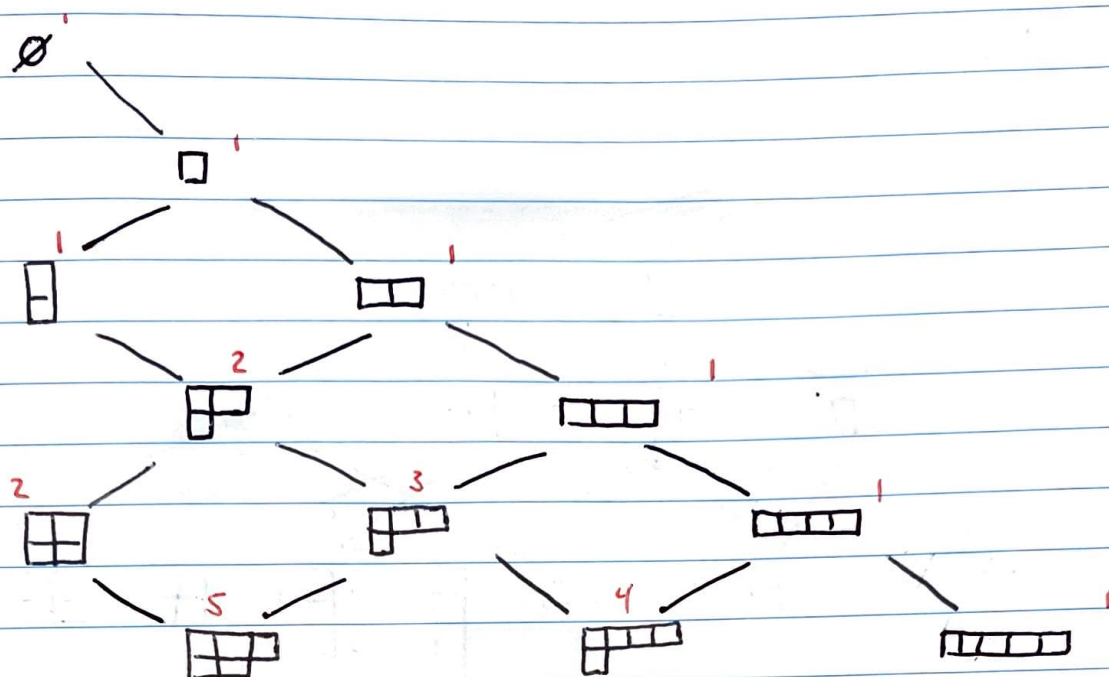
$$(3) \quad 3^2 + 1^2 + 2^2 + 1^2 = 15$$

$$(4) \quad 3^2 + 6^2 + 6^2 + 1^2 + 3^2 + 2^2 + 3^2 + 1^2 = 105$$

1, 3, 15, 105, ...

These are the odd double factorial numbers: $(2k-1)!!$

Temperley-Lieb (Restrict to two rows)



We have:

$$(0) \quad 1^2 = 1$$

$$(1) \quad 1^2 = 1$$

$$(2) \quad 1^2 + 1^2 = 2$$

$$(3) \quad 2^2 + 1^2 = 5$$

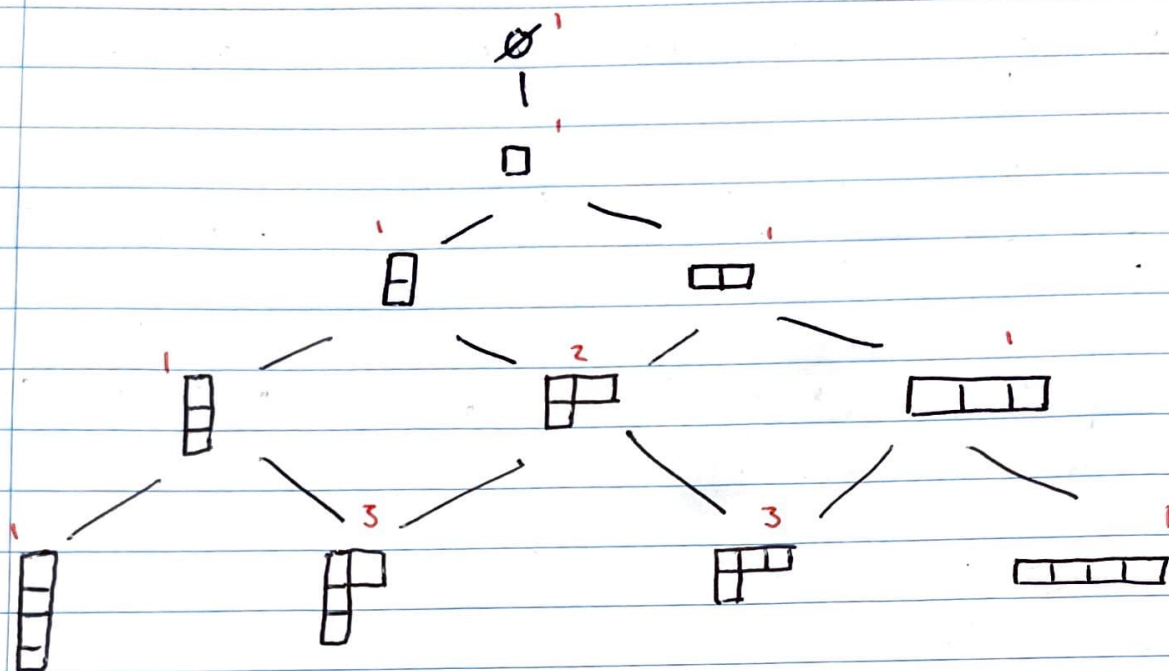
$$(4) \quad 2^2 + 3^2 + 1^2 = 14$$

$$(5) \quad 5^2 + 4^2 + 1^2 = 42$$

1, 1, 2, 5, 14, 42

These are the Catalan numbers: $\frac{1}{k+1} \binom{2k}{k}$

Pascal's triangle (restrict to along the wall)



We have:

$$(0) \quad 1^2 = 1$$

$$(1) \quad 1^2 = 1$$

$$(2) \quad 1^2 + 1^2 = 2$$

$$(3) \quad 1^2 + 2^2 + 1^2 = 6$$

$$(4) \quad 1^2 + 3^2 + 3^2 + 1^2 = 20$$

These are binomial coefficients of the form: $\binom{2k}{k}$.